Randomness Analysis of Gioia Systems *Cut N' Shuffle* Card Shuffler

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Introduction

Cut N' Shuffle is a card shuffler developed by Gioia Systems, LLC, 650 South Cherry Street, Suite 500, Denver, Colorado 80246-1803, Gene Gioia, Chief Executive Officer. The purpose of this report is to provide a mathematical analysis on the randomness of the shuffles produced by the *Cut N' Shuffle*. Pre- versus post-shuffle charts and a variety (more than half a dozen) of statistical tests are used to assess the randomness of the shuffles. The pre- versus post-shuffle charts reveal no obvious patterns and the *Cut N' Shuffle* passed each of the tests for randomness. Based on the data and analysis conducted herein we conclude that *Cut N' Shuffle* produces random shuffles.

Data and Methodology

A random shuffle of 52 cards is one in which all 52! possible orderings of the cards are equally likely. A shuffle of 52 cards can be considered a permutation of the integers {1, 2, ..., 52}, and a shuffling procedure is a process which produces such permutations. More formally, a shuffle procedure is a probability distribution defined on S_{52}, where S_{52} is the set of all possible permutations of {1, 2, ..., 52}. A shuffling process is random if it is equivalent to the uniform distribution on S_{52}.

Confirming randomness is complicated due to the many possible alternatives to the null hypothesis of a random process. Consequently, no single test is powerful against all types of departures from randomness and a variety of statistical tests are needed to verify randomness. Showing that a process such as a shuffle is random entails applying numerous statistical tests (typically about half a dozen) to data generated from this process and seeing if the outcomes from the process pass these tests. To "pass" a statistical test for randomness means that we fail to reject the null hypothesis of randomness. If the shuffle process passes these tests for randomness satisfactorily[1] we considered it to be random – it is then presumed innocent until proven guilty.[2]

The *Cut N' Shuffle* device is designed to shuffle each of two single decks (herein Deck 1 and Deck 2) of 52 playing cards on each shuffle cycle. The data in this study consists of the pre- and post-shuffle position of each of the 52 cards for 400 shuffles, 200 for Deck 1 and 200 for Deck 2. That is, for 200 shuffles each of Decks 1 and 2, the cards were ordered from 1 to 52 prior to each shuffle and the post-shuffle position recorded for each card. The analyses included pre- versus post-shuffle plots,[3] concordance among shuffles for individual cards (positions) and for ¼-deck segments, chi-squared tests for pre- versus post-shuffle tables of ¼-deck counts, analysis of variance and Kruskal-Wallis tests for the difference in average post-shuffle positions of individual cards and ¼-deck segments of cards, rising sequence analysis, gap analysis, and clump analysis. The results of these analyses are detailed below.

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[1] We use a nominal .10 level of significance; the *Cut N' Shuffle* passes many tests at a level much greater than .10.
[2] Donald E. Knuth (1998), *The Art of Computer Programming*, Vol. 2, Seminumerical Algorithms, 3rd. ed.
[3] In this report we display 72 plots, one dozen each from the beginning, middle, and end of the data set of 200 shuffles for each of Deck 1 and Deck 2.

Results

Pre- versus Post Shuffle Plots

Figures 1.1 through 1.36 and 2.1 through 2.36 in the Appendix show the plots of pre- vs. post-shuffle position for 36 selected shuffles for each of Decks 1 and 2.[4] For comparison purposes several plots showing non-random shuffles have been included in Appendix 2. The non-random plots show the patterns for shuffles in which: (a) the cards have not been moved (i.e., no shuffle), (b) the order of the cards has merely been reversed, (c) a simple cut was made (the bottom half of the cards has been moved to the top half), (d) the positions of adjacent cards have been interchanged, (d) ¼-deck segments have been preserved, and (e) 4-card clumps have been preserved. Note that in each of these non-random shuffles there is a readily apparent pattern in the plot.[5] A mathematically random shuffle will tend to produce graphs with no apparent pattern. The *Cut N' Shuffle* plots exhibit no obvious patterns and suggest that the shuffles are random. The detailed statistical analyses that follow will further support this conclusion.

Concordance among Shuffle Iterations

Kendall's coefficient of concordance *W* is a measure of agreement among repeated iterations of a shuffle. This concordance measure can range between 0 and 1, with values near 1 indicating a high level of agreement, and can be interpreted roughly as a multiple correlation coefficient. A random shuffle would tend to produce a concordance coefficient among repeated iterations near zero.[6] Average (mean) post-shuffle positions for each of the 52 pre-shuffle cards (positions) were computed for each of the 200 iterations each of Decks 1 and 2, and Kendall's coefficient of concordance calculated for these post-shuffle averages. The results are as follows.

Concordance Analysis – individual cards (positions)		
	Deck 1	*Deck 2*
Concordance Coefficient *W*	$W = 0.006$	$W = 0.005$
Average Spearman Correlation r_s	$r_s = .0009$	$r_s = -.00005$
Chi-squared Statistic (51 df)	$\chi^2 = 59.77$	$\chi^2 = 50.54$
p-value	$p = .187$	$p = .492$
Conclusion	Not significant	Not significant

The observed concordance coefficients are near zero and not statistically significant at the .10 level for both Decks, indicating no agreement among the 200 shuffles. The average of the Spearman correlations between pairs of shuffles is also near zero for both Decks. A similar analysis on the average post-shuffle positions for ¼-deck segments yields the following results.

[4] Figures 1.1 through 1.12 are shuffles 1-12 of Deck 1; Figures 1.13 through 1.24 are shuffles 95-106 of Deck 1; Figures 1.25 through 1.36 are shuffles 189-200 of Deck 1; Figures 2.1 through 2.12 are shuffles 1-12 of Deck 2; Figures 2.13 through 2.24 are shuffles 95-106 of Deck 2; Figures 2.25 through 2.36 are shuffles 189-200 of Deck 2.

[5] See Hannum (2000) for examples of actual six-deck card shuffles used in casinos which are not random. This study is also discussed, though not as thoroughly, in Hannum and Cabot (2005).

[6] The interested reader is referred to standard texts on nonparametric statistics, such as the one by Siegel and Castellan (1988), for further discussion on the coefficient of concordance.

Concordance Analysis – ¼-deck segments		
	Deck 1	*Deck 2*
Concordance Coefficient *W*	$W = 0.009$	$W = 0.003$
Average Spearman Correlation r_s	$r_s = .004$	$r_s = –.002$
Chi-squared Statistic (3 df)	$\chi^2 = 5.33$	$\chi^2 = 1.67$
p-value	$p = .149$	$p = .643$
Conclusion	Not significant	Not significant

Again the concordance coefficients are near zero and not significant at the .10 level of significance, and the average Spearman correlations are also near zero, reflecting no agreement among the 200 shuffles for both Decks 1 and 2.

Pre- vs. Post-Shuffle Tables

For each of the two Decks, a combined two-way table of post-shuffle ¼-deck packs against pre-shuffle ¼-deck packs was constructed. The resulting counts are shown below.

Pre- vs. Post-Shuffle Positions – Deck 1 (¼-decks)				
	PostQ1	*PostQ2*	*PostQ3*	*PostQ4*
PreQ1	649	676	634	641
PreQ2	638	622	660	680
PreQ3	670	652	645	633
PreQ4	643	650	661	646

Pre- vs. Post-Shuffle Positions – Deck 2 (¼-decks)				
	PostQ1	*PostQ2*	*PostQ3*	*PostQ4*
PreQ1	640	615	662	683
PreQ2	660	674	624	642
PreQ3	659	636	671	634
PreQ4	641	675	643	641

A random shuffle should produce roughly equal counts (in this case, 650) in each of the sixteen cells. The counts in the above tables are not too different (as one could be deduced from the pre- vs. post-shuffle plots previously discussed) and in fact are not statistically significantly different at the .10 level of significance as seen from the following chi-squared test results.

Chi-Squared Table Analysis – ¼-deck segments		
	Deck 1	*Deck 2*
Chi-squared Statistic (9 df)	$\chi^2 = 5.92$	$\chi^2 = 8.90$
p-value	$p = .748$	$p = .447$
Conclusion	Not significant	Not significant

The above chi-squared test results confirm that the counts in the 4x4 tables for Decks 1 and 2 do not differ significantly from one another, suggesting that the shuffles are random.

Analysis of Variance and Kruskal-Wallis Tests

The Analysis of Variance (ANOVA) and Kruskal-Wallis tests are procedures which can answer whether the average post-shuffle positions of each of the 52 cards (positions) are significantly different from one another. The former is a standard "normal theory" test comparing the mean post-shuffle positions while the latter is a nonparametric procedure comparing the median post-shuffle positions. A truly random shuffle would tend to produce average post-shuffle positions which are roughly equal. The results of these tests for Decks 1 and 2 are as follows.

Analysis of Variance – individual cards (positions)		
	Deck 1	*Deck 2*
ANOVA *F* Statistic (51 & 10,348 df)	$F = 1.20$	$F = 1.01$
p-value	$p = .160$	$p = .452$
Conclusion	Not significant	Not significant

Kruskal Wallis Test – individual cards (positions)		
	Deck 1	*Deck 2*
Kruskal-Wallis Statistic ($\approx \chi^2$ with 51 df)	$KW = 60.95$	$KW = 51.55$
p-value	$p = .160$	$p = .452$
Conclusion	Not significant	Not significant

The ANOVA table above shows that there is no significant difference at the .10 level among the mean post-shuffle positions of the 52 pre-shuffle cards (positions) for both Deck 1 and Deck 2, as reflected by the *p*-values of .160 and .452, respectively. The Kruskal-Wallis test produces similar results with the same .160 and .452 *p*-values. These tests suggest that the shuffles are random.

A similar analysis was conducted to compare the average post-shuffle positions of each ¼-deck segment of cards. The results are as follows.

Analysis of Variance – ¼ deck segments		
	Deck 1	*Deck 2*
ANOVA *F* Statistic (3 & 796 df)	$F = 1.53$	$F = 2.02$
p-value	$p = .204$	$p = .110$
Conclusion	Not significant	Not significant

Kruskal Wallis Test – ¼ deck segments		
	Deck 1	*Deck 2*
Kruskal-Wallis Statistic ($\approx \chi^2$ with 3 df)	$KW = 4.75$	$KW = 4.25$
p-value	$p = .191$	$p = .236$
Conclusion	Not significant	Not significant

The ANOVA and Kruskal-Wallis tests applied to ¼-deck segments are also not significant at the .10 level for both Decks 1 and 2, supporting the hypothesis that the average post-shuffle positions for the ¼-deck packs are not different and suggesting the shuffles are random.

Rising Sequences

A rising sequence in a permutation is a maximal consecutively increasing subsequence. For example, the permutation 253146 has three rising sequences (1, 234, 56), the permutation 314265 also has three rising sequences (12, 345, 6), and the permutation 426135 has four rising sequences (1, 23, 45, 6). Applying this notion to a deck of cards, an ordered deck of *n* cards has one rising sequence, while the permutation corresponding to a complete reversal of the deck would contain *n* rising sequences.[7] The number of permutations of *n* objects (cards) with exactly *r* rising sequences is given by the Eulerian number $A_{n,r}$, where $A_{n,r}$ can be defined by the recursive relationship

$$A_{n+1,r} = (n-r+2)A_{n,r-1} + rA_{n,r}$$

with $A_{0,0} = 1$ and $A_{n,0} = 0$ for $n>0$.[8] Due to the asymptotic normality of the Eulerian numbers, if *R* represents the number of rising sequences in a random shuffle of *n* cards, then for large *n*, the distribution of *R* is approximately normal with $E(R) = (n+1)/2$ and $V(R) = (n+1)/12$. For one random shuffle of a single deck (n=52), then, the expected number of rising sequences is $E(R) = 26.5$, with standard deviation $SD(R) = 2.10$. For a random sample of *m* values of *R*, each from a random shuffle of *n* cards, the expected value of the average *R*-value, $\bar{R}$, is $(n+1)/2$ with variance $(n+1)/12m$. With 200 shuffles of a single deck, then, the expected value and standard deviation of the average *R*-value are $E(\bar{R}) = 26.5$ and $SD(\bar{R}) = 0.149$, respectively. The rising sequences results for the 200 shuffles of each of Decks 1 and 2 are provided in the table below.

Rising Sequences		
	Deck 1	*Deck 2*
Average Rising Sequences	$\bar{R} = 26.60$	$\bar{R} = 26.51$
Z Statistic	$z = 0.639$	$z = 0.067$
p-value	$p = .523$	$p = .946$
Conclusion	Not significant	Not significant

For both Deck 1 and Deck 2 the average number of rising sequences is very close to the expected value of 26.5. Both *p*-values exceed .50 indicating the results are not significant. The rising sequences test clearly suggests the shuffles for Decks 1 and 2 are random.

Gaps between Initially Adjacent Cards

The post-shuffle gap between initially adjacent cards *i* and *i*+1 was calculated by counting forward from the post-shuffle position of card *i* until reaching card *i*+1, and assuming position 1 followed the card in position 52. Thus, if initially adjacent cards were adjacent after the shuffle but in reverse order (for example, if the post-shuffle positions of cards 1 and 2 were 35 and 34 respectively), the gap would equal 50. Letting *G* represent the gap, as defined above, between initially adjacent cards, then for a random shuffle of *n* cards, *G* has a uniform distribution on $\{0, 1, ..., n-2\}$and the expected gap size is $E(G) = (n-2)/2$. For a random shuffle of a single deck of

[7] Bayer and Diaconis (1992) used the notion of rising sequences in their argument that seven riffle shuffles are required to thoroughly randomize a deck of 52 cards.

[8] The interested reader is referred to Carlitz et al (1972), Tanny (1973), and the references cited therein for further details regarding Eulerian numbers.

52 cards, then, G is uniform on {0, 1, ..., 50}, with expected gap size of $E(G) = 25$. The table below shows the gap analysis results for the 200 shuffles of Decks 1 and 2.

Gap Analysis		
	Deck 1	*Deck 2*
Average Gap Size	$\bar{G} = 25.08$	$\bar{G} = 24.98$
Chi-squared Statistic (50 df)	$\chi^2 = 58.90$	$\chi^2 = 45.43$
p-value	$p = .182$	$p = .657$
Conclusion	Not significant	Not significant

The average gap size for Decks 1 and 2 are 25.08 and 24.98, respectively, very close to the expected value of 25.0 for a random shuffle. The chi-squared value is the test statistic for a goodness-of-fit test applied to the observed gap frequencies. If a shuffle is random, then G is uniform on {0, 1, ..., 50} and so one would expect equal frequencies for each of the 51 possible gap sizes. For 200 iterations of a random shuffle, then, the expected frequency for each of the 51 gap sizes is 200. The goodness-of-fit test looks at how different each of the observed gap frequencies is from the expected value of 200. If this difference is large (aggregated across the 51 gap sizes), it suggests a non-random shuffle; if this difference is small, it suggests a random shuffle. The results in the gap analysis table above show that the chi-squared goodness-of-fit tests are non-significant at the .10 level for both Deck 1 and Deck 2, with *p*-values of .182 and .657, respectively.

Clumps

A clump is a maximal group of consecutive cards that remain together, in the same relative order but not necessarily in the same collective location, through the shuffle. The following table shows the frequencies and percentages of clumps of cards after the 200 shuffles for each of Decks 1 and 2.

Clump Analysis				
	Deck 1		*Deck 2*	
	Frequency	Percent	Frequency	Percent
1-Card	10,048	98.29%	10,048	98.10%
2-Card	173	1.69%	173	1.86%
3-Card	2	0.02%	2	0.04%
More	0	0.0%	0	0.0%
	10,223	100.00%	10,202	100.00%

The clump percentages in the table above show no large clumps of cards remain together after the shuffles with less than 2% two-card clumps, less than 0.1% three-card clumps, and no clumps of more than three cards. These results are consistent with a random shuffle and better than those found in one study of professional dealers. Hannum (2000) found 4% two-card clumps in a study of a seven-riffle single-deck shuffle – the shuffle shown by Bayer and Diaconis (1992) to be a random shuffle – and Epstein (1995) reported 80% one-card, 18% two-card, and 2% three-or-more-card clumps for professional dealers.

Summary and Conclusion

This report provides a mathematical analysis of the randomness of shuffles produced by Gioia Systems *Cut N' Shuffle* device. Methods used to assess the randomness of the shuffles include pre- versus post-shuffle plots and a variety of statistical tests. In analyzing over four-hundred shuffles – two hundred for each of the two deck positions in the *Cut N' Shuffle* device – we find that the shuffles produced are random, passing all tests for mathematical randomness. The plots and statistical tests are consistent and unequivocal in showing no evidence that the shuffles are not random. The conclusion is clear: shuffles produced by *Cut N' Shuffle* result in a random rearrangement of cards.

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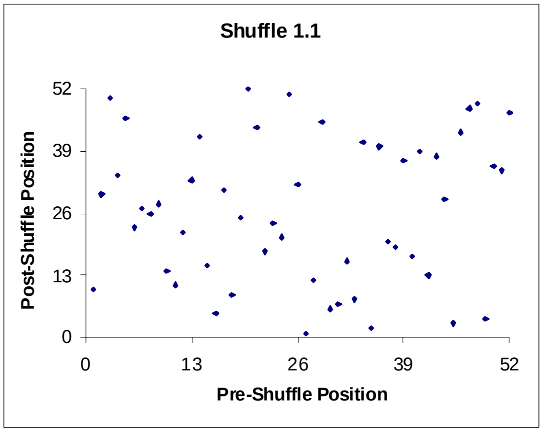
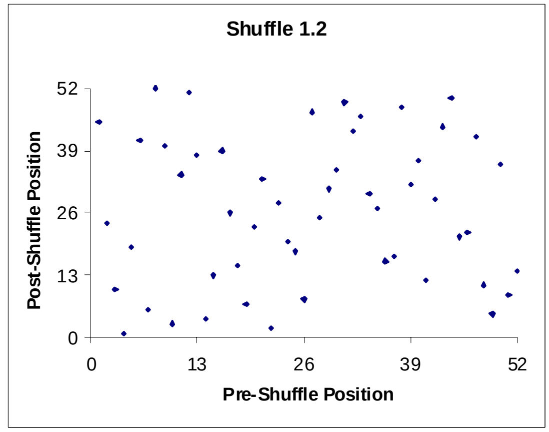
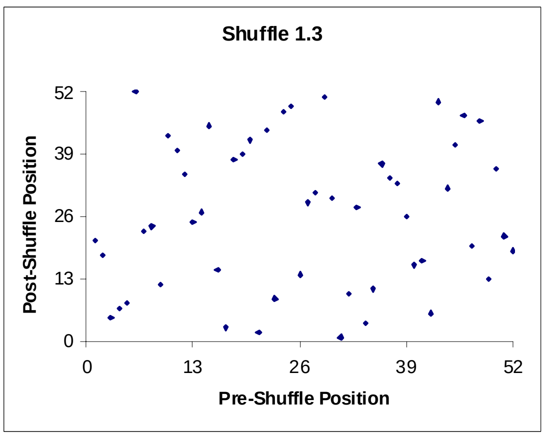
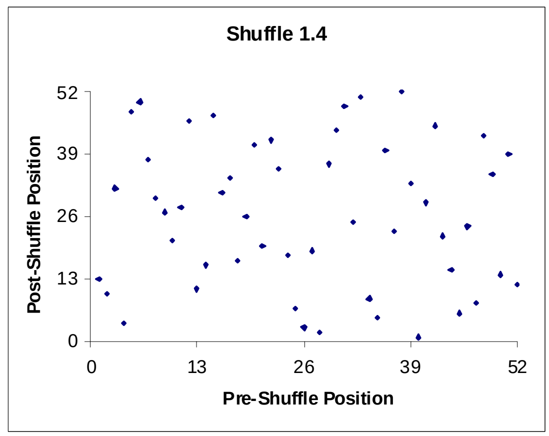
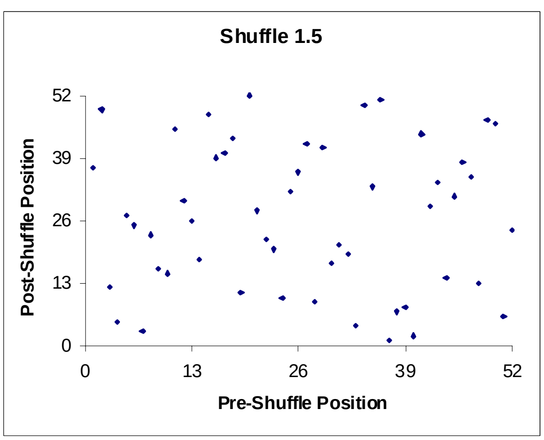
Appendix 1 – *Cut N' Shuffle* Plots

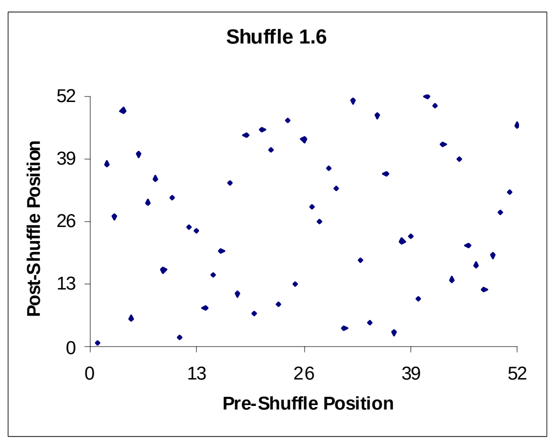


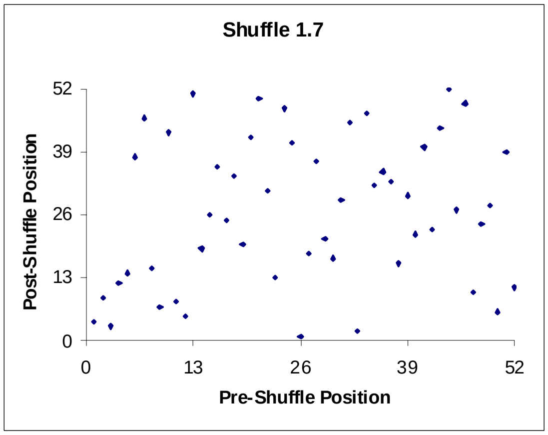












Shuffle 1.7
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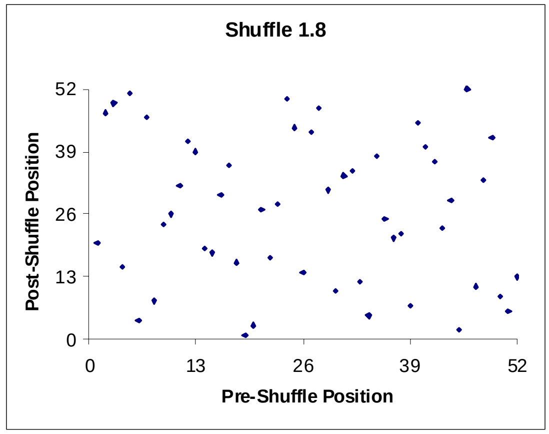
39
26
13
0
0
13
26
39
52
Post-Shuffle Position
Pre-Shuffle Position



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13
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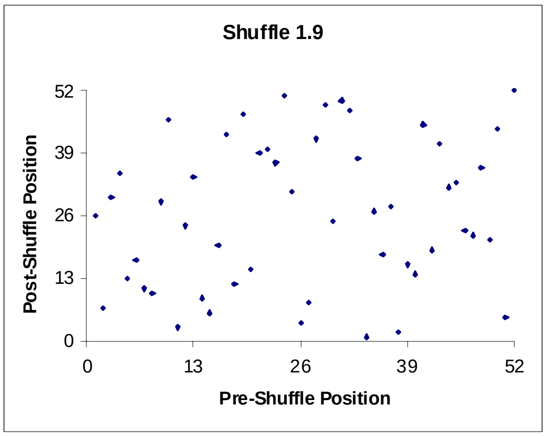
52
Post-Shuffle Position
Pre-Shuffle Position



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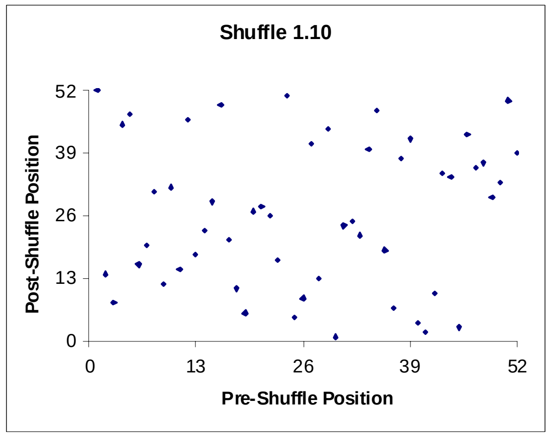
26
13
0
0
13
26
39
52
Post-Shuffle Position
Pre-Shuffle Position



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13
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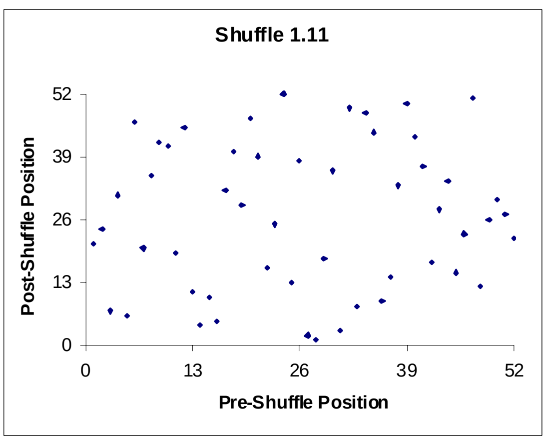
0
13
26
39
52
Post-Shuffle Position
Pre-Shuffle Position



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13
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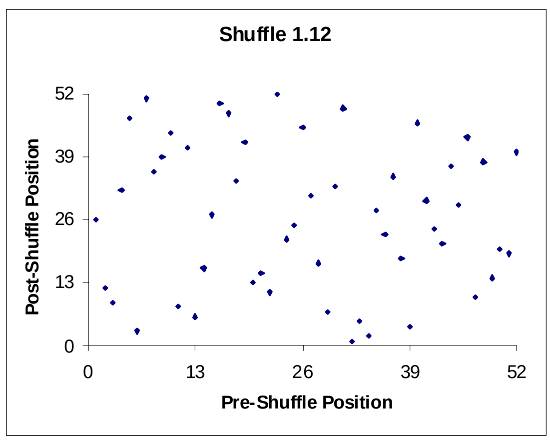
0
13
26
39
52
Post-Shuffle Position
Pre-Shuffle Position



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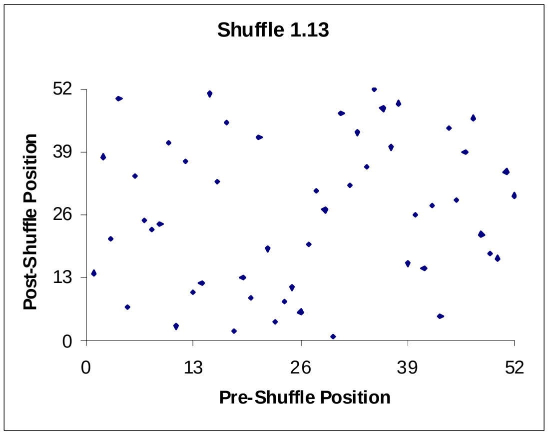
52
Post-Shuffle Position
Pre-Shuffle Position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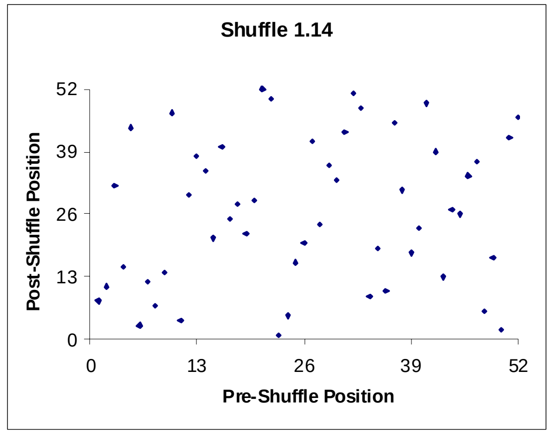
13
26
39
52
Post-Shuffle Position
Pre-Shuffle Position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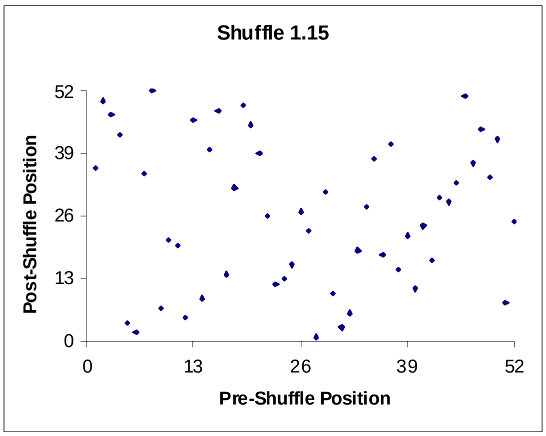
Post-Shuffle Position
Pre-Shuffle Position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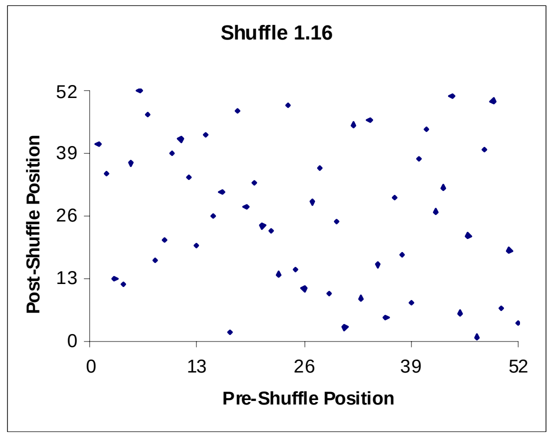
Pre-Shuffle Position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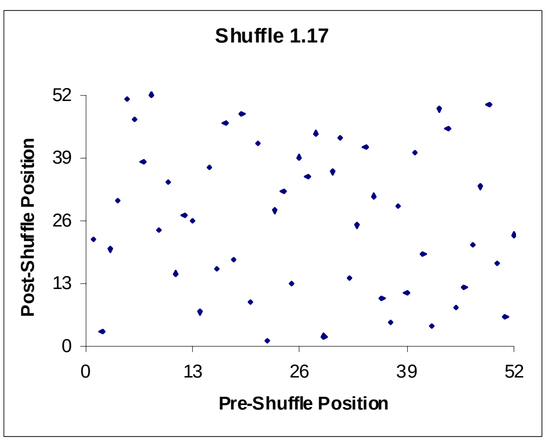
Post-Shuffle Position
Pre-Shuffle Position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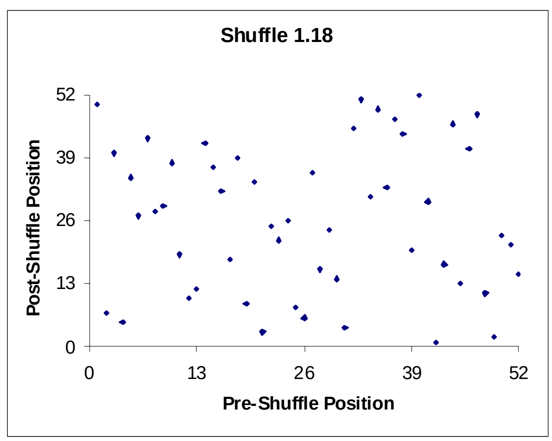
52
Post-Shuffle Position
Pre-Shuffle Position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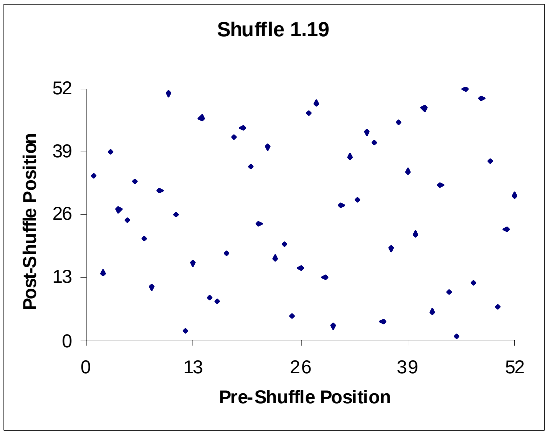
Post-Shuffle Position
Pre-Shuffle Position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Post-Shuffle Position
Pre-Shuffle Position
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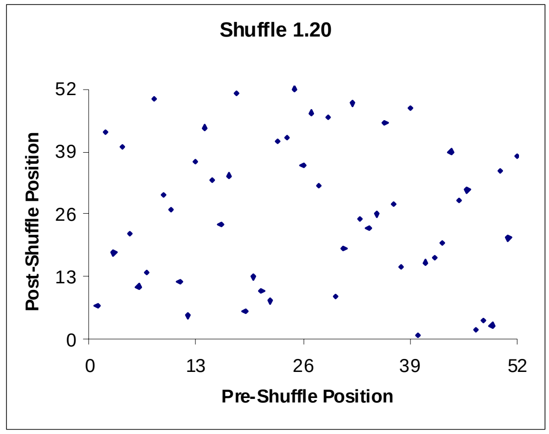
26
39
52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Post-Shuffle Position
Pre-Shuffle Position
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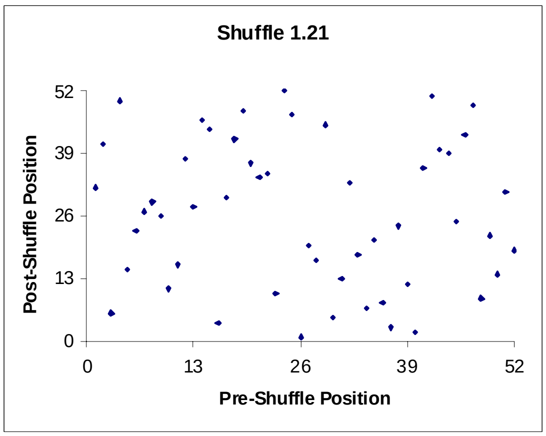
26
39
52


Shuffle 1.21
Post-Shuffle Position
Pre-Shuffle Position
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13
26
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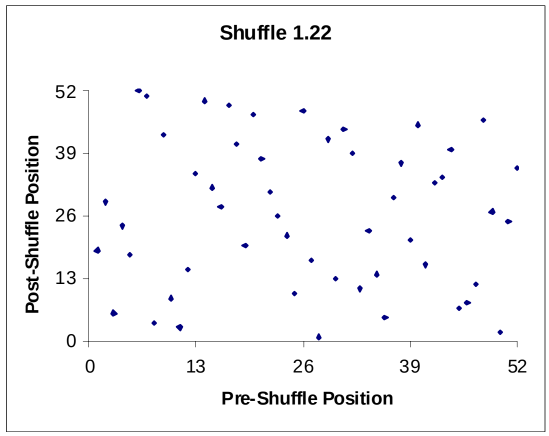
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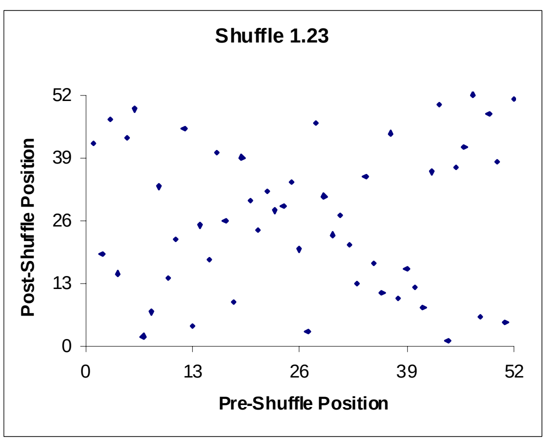
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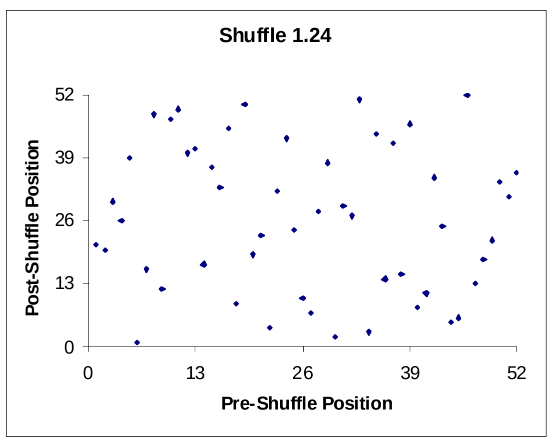
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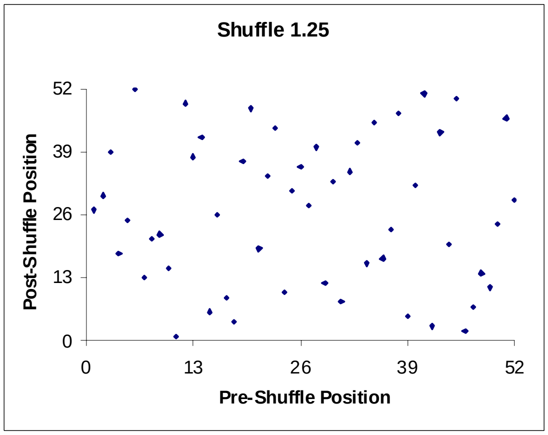
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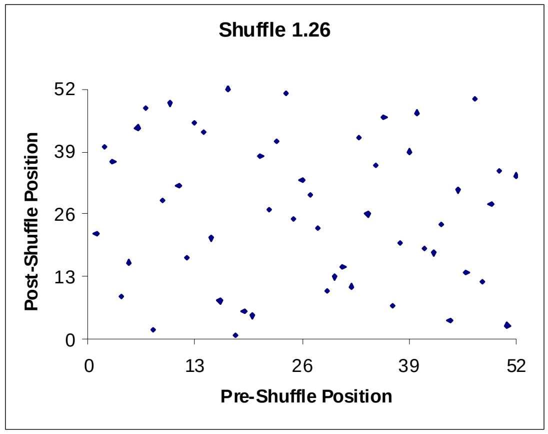
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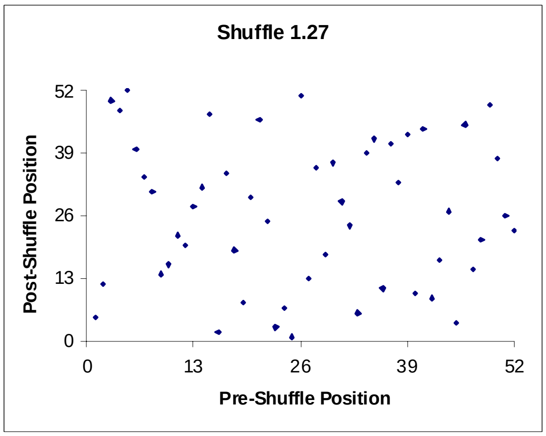
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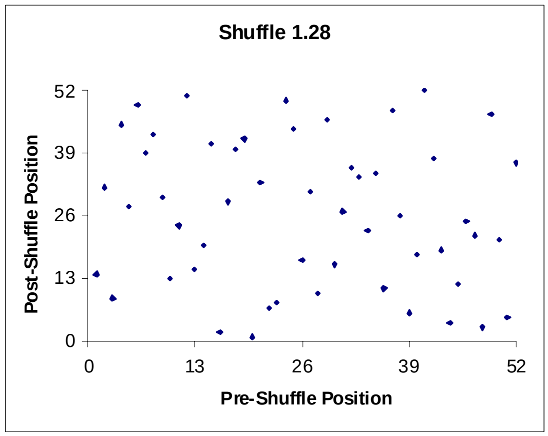
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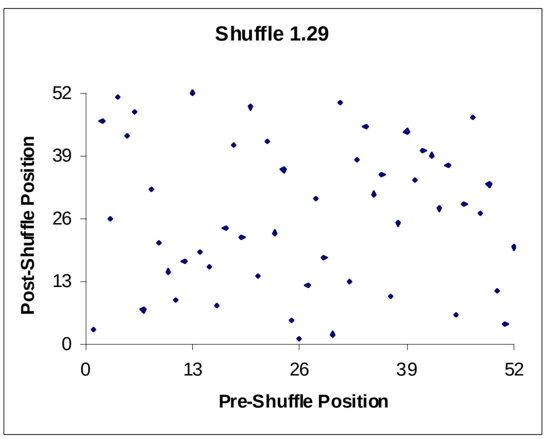
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Post-Shuffle Position
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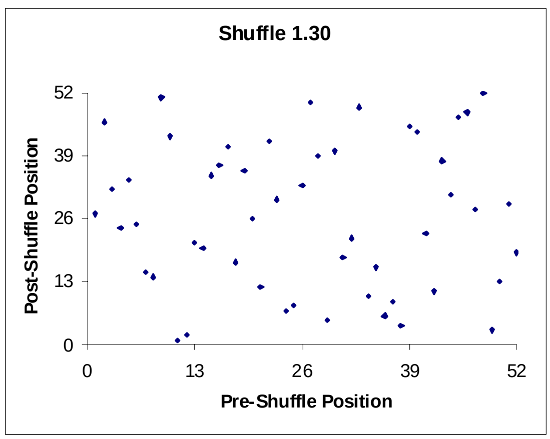
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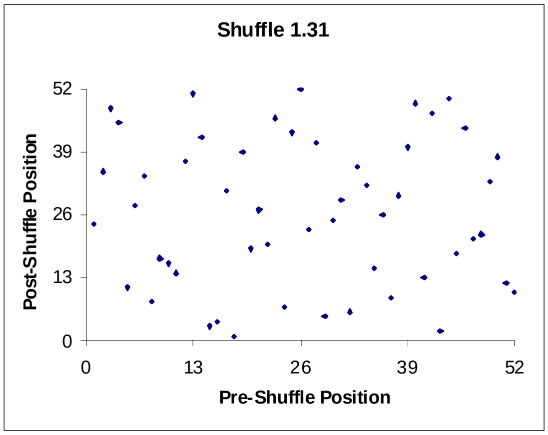
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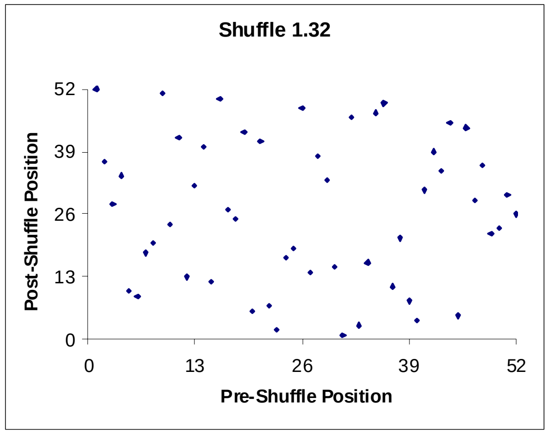
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Post-Shuffle Position
Pre-Shuffle Position
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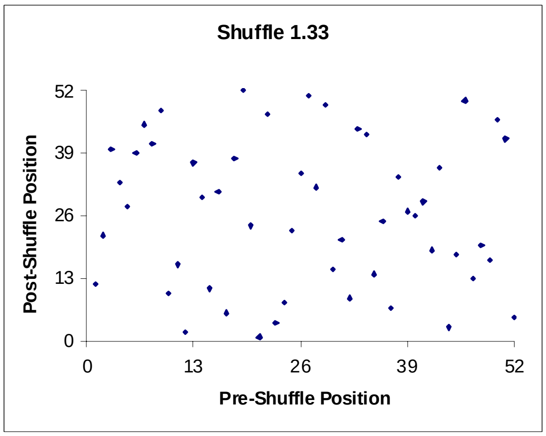
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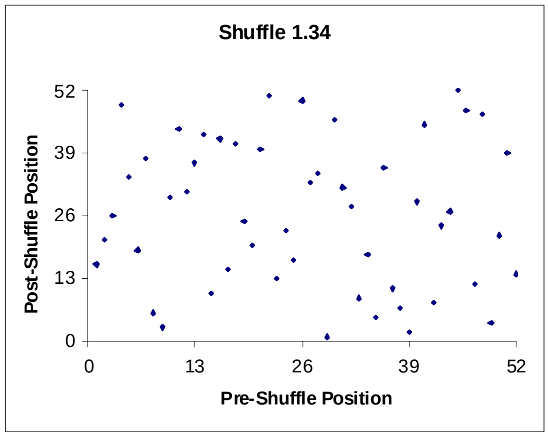
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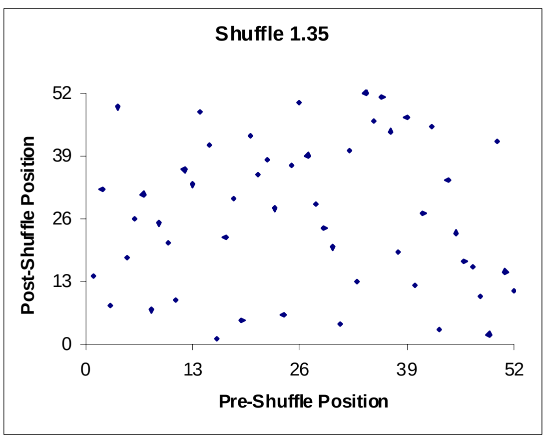
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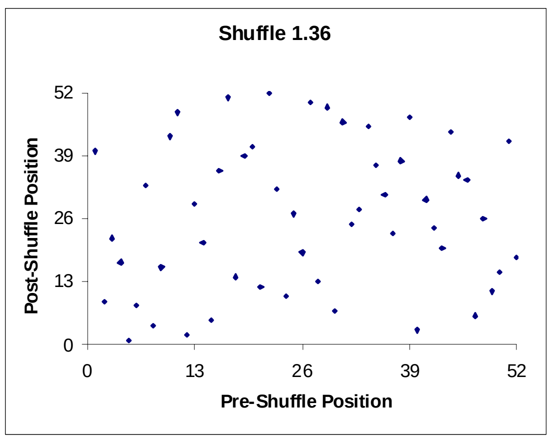
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Post-Shuffle Position
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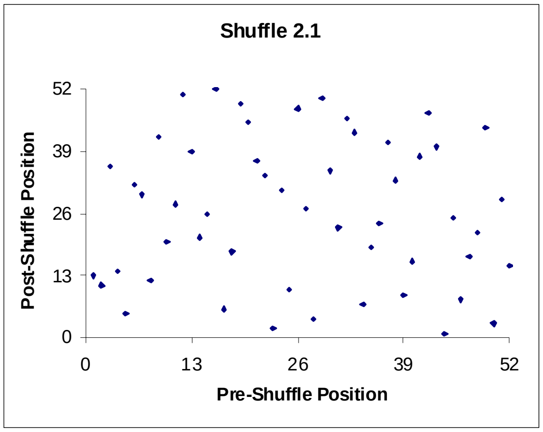
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Post-Shuffle Position
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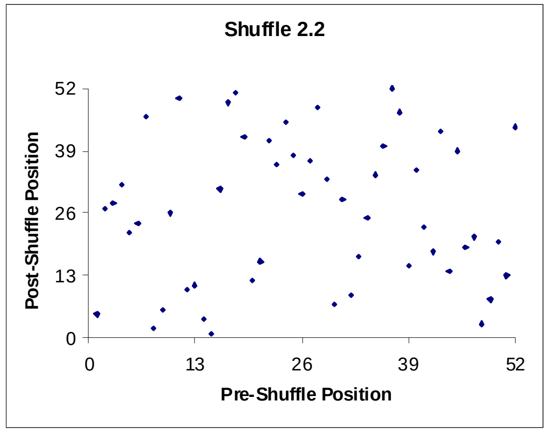
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Post-Shuffle Position
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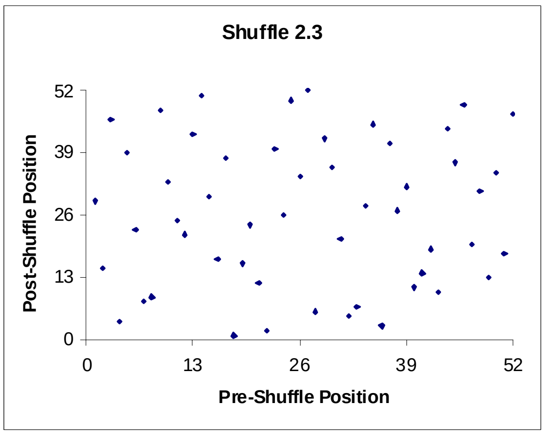
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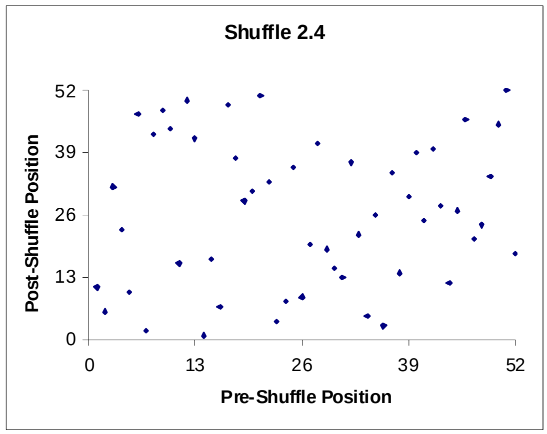
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Post-Shuffle Position
Pre-Shuffle Position
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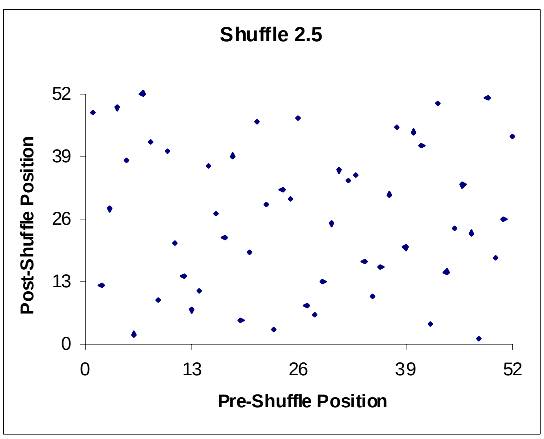
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Post-Shuffle Position
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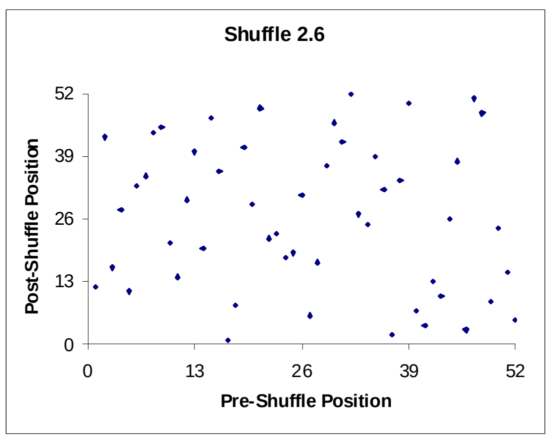
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Post-Shuffle Position
Pre-Shuffle Position
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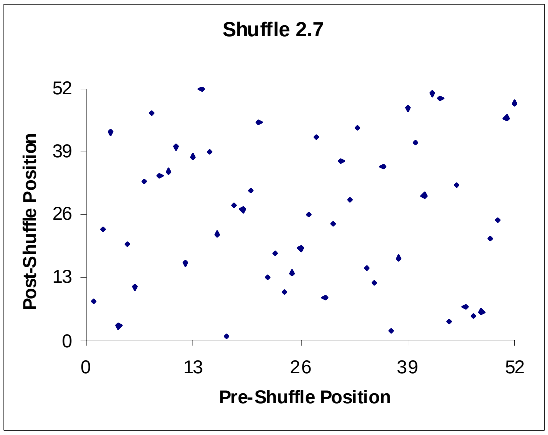
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Post-Shuffle Position
Pre-Shuffle Position
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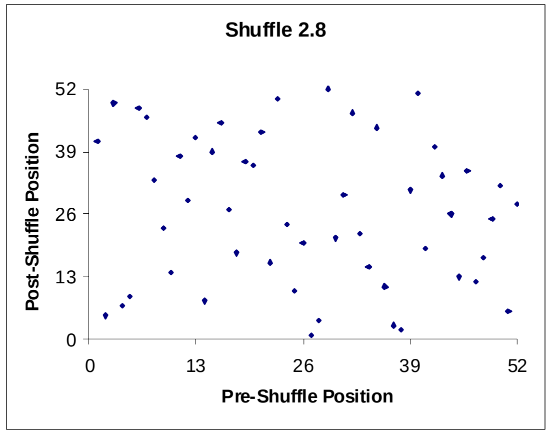
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Post-Shuffle Position
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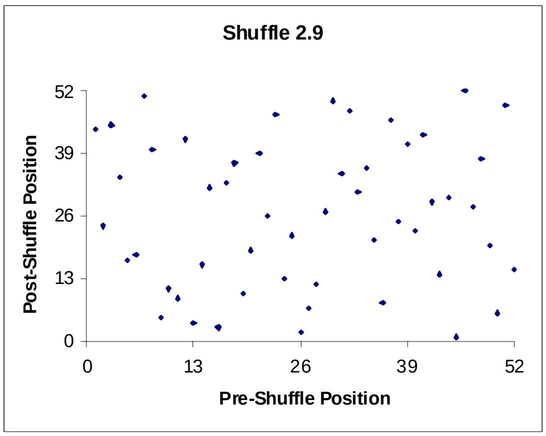
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Post-Shuffle Position
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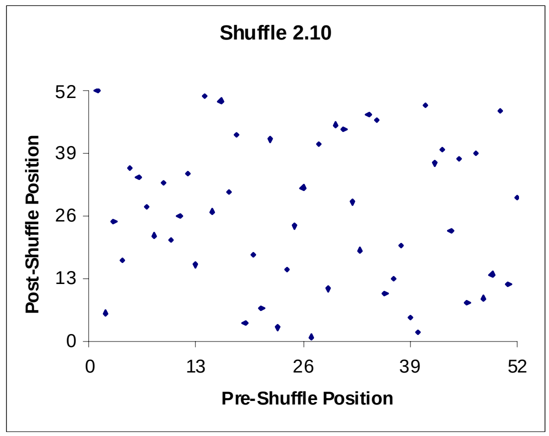
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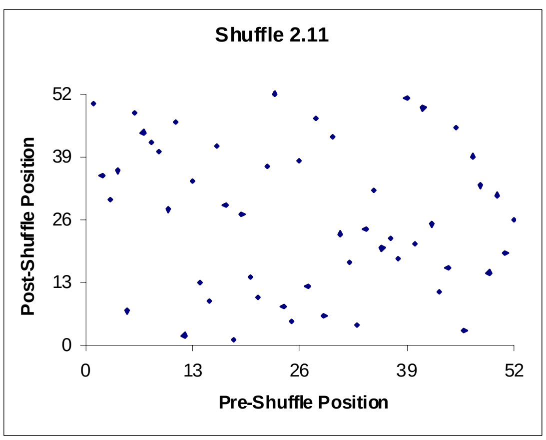
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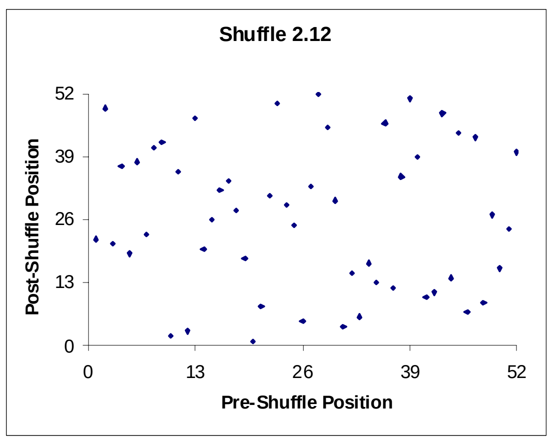
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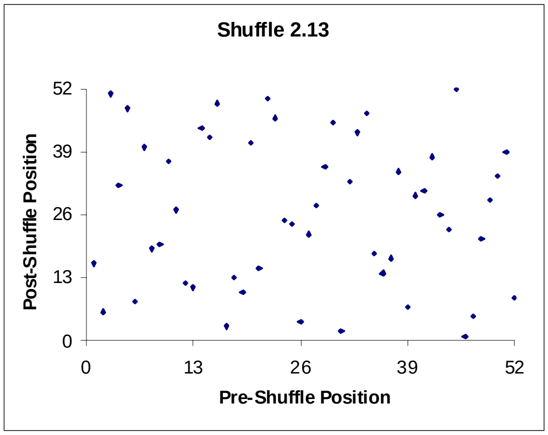
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Post-Shuffle Position
Pre-Shuffle Position
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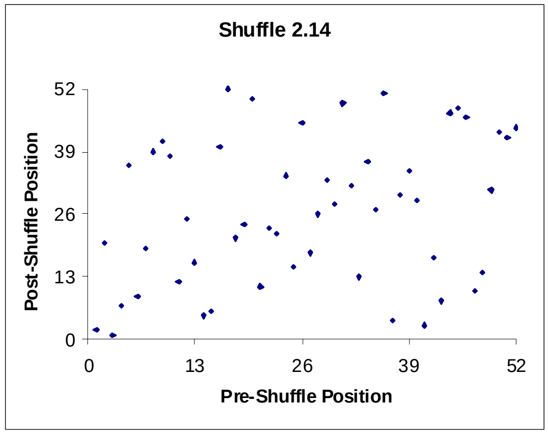
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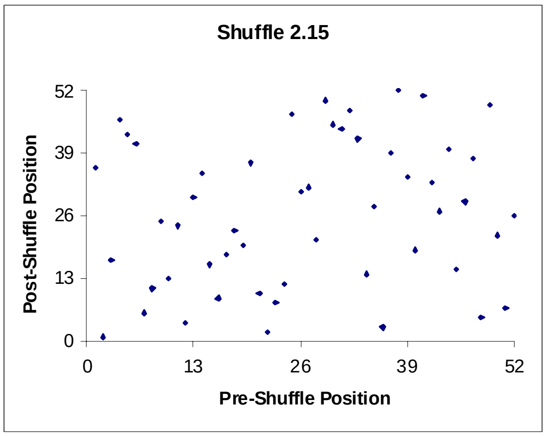
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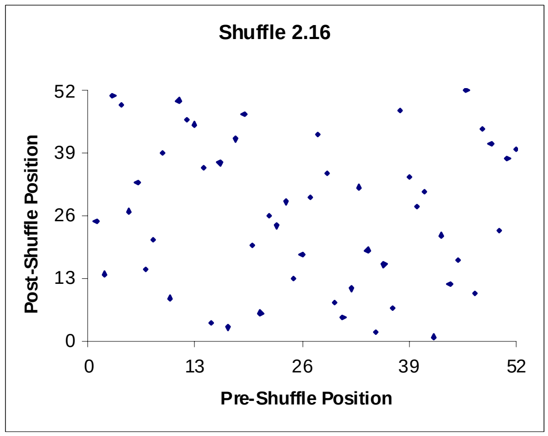
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Post-Shuffle Position
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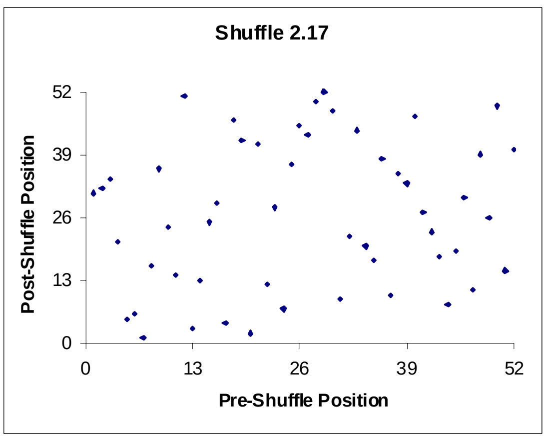
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Post-Shuffle Position
Pre-Shuffle Position
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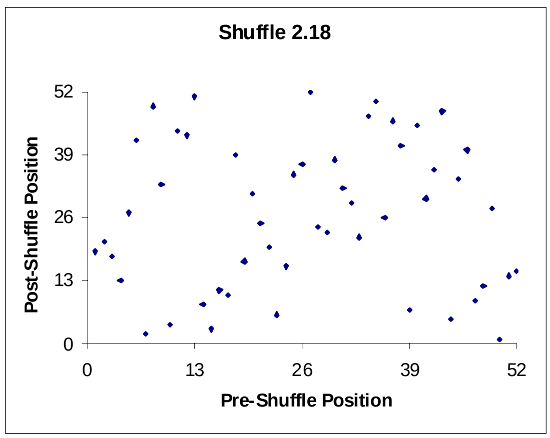
52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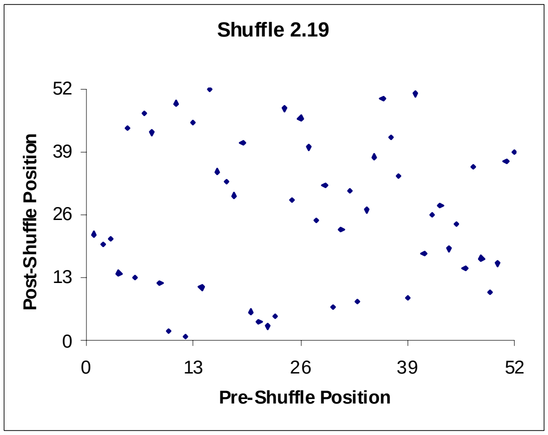
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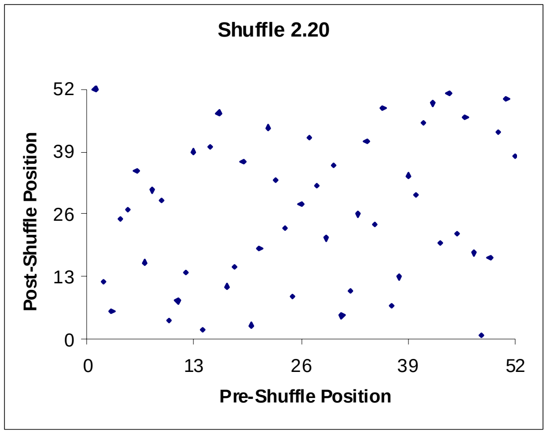
Pre-Shuffle Position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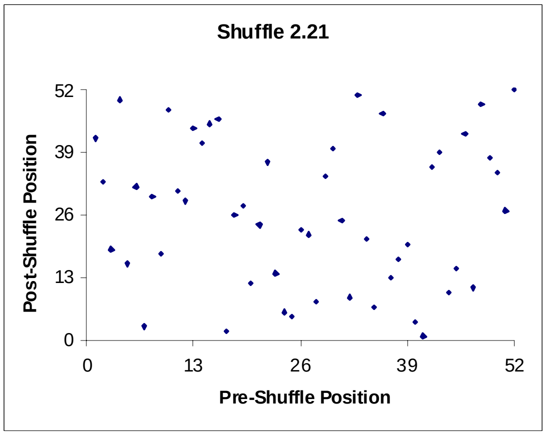
Pre-Shuffle Position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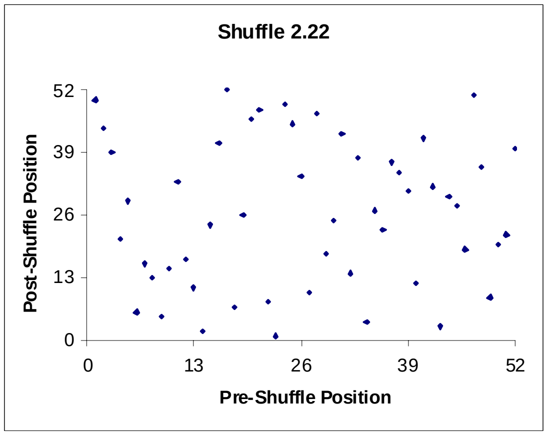
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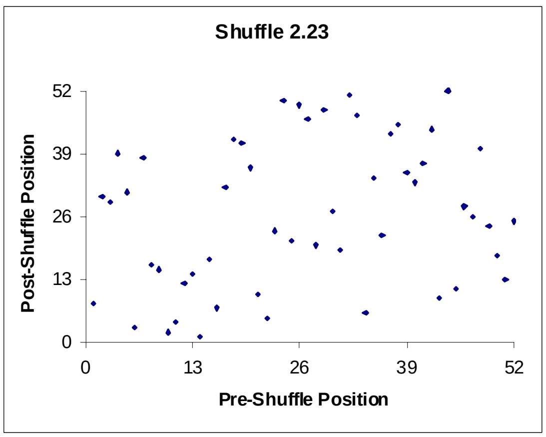
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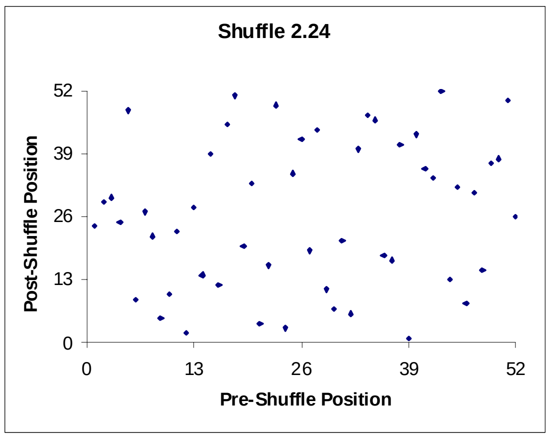
Pre-Shuffle Position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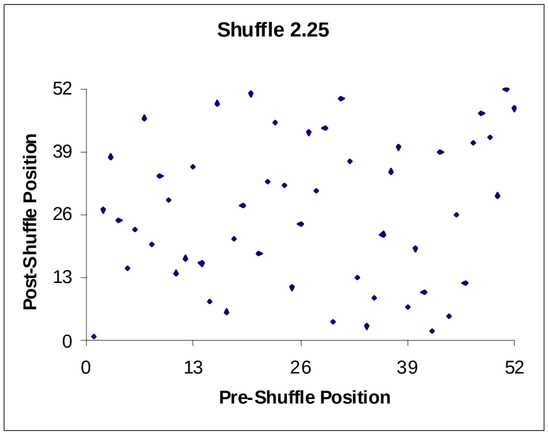
Pre-Shuffle Position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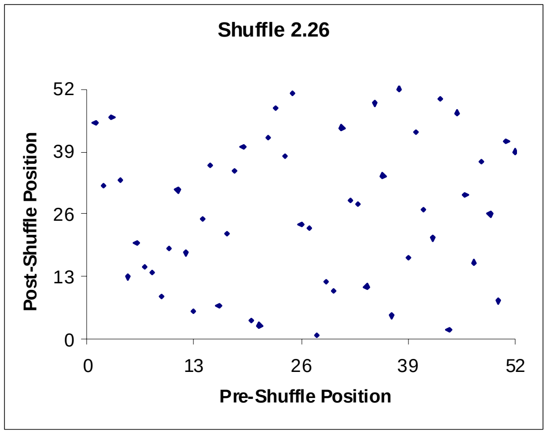
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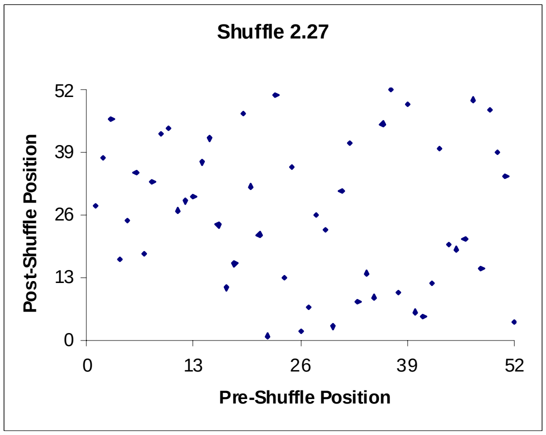
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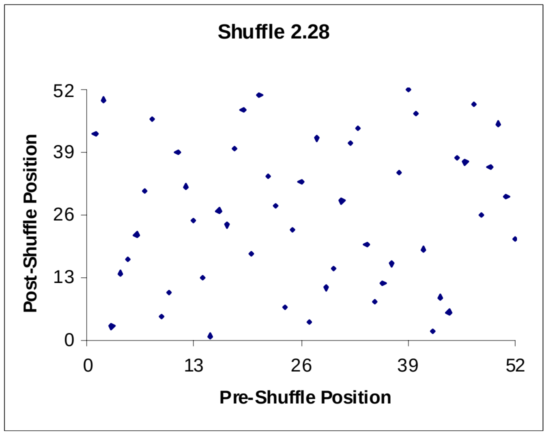
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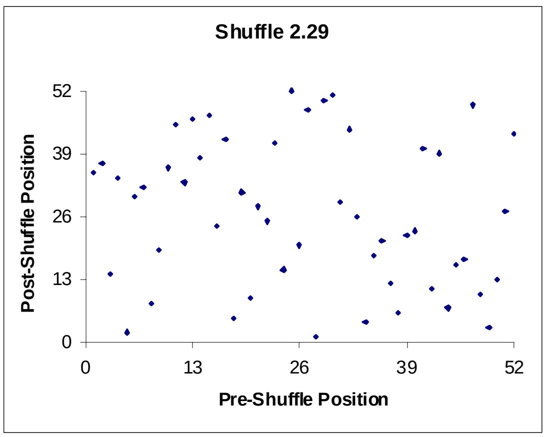
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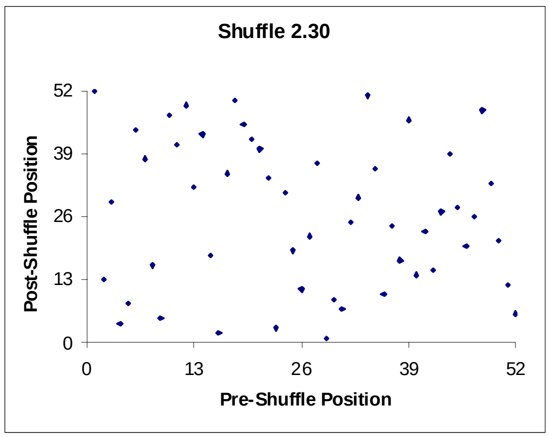
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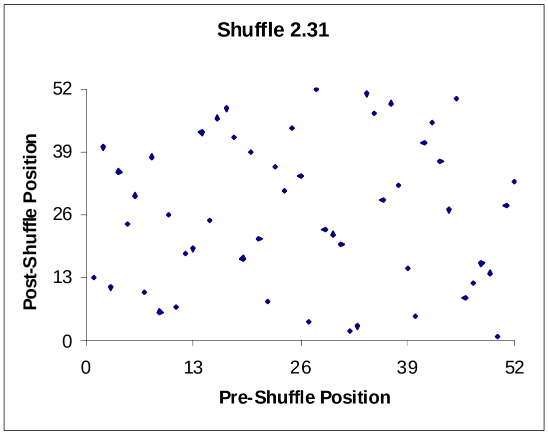
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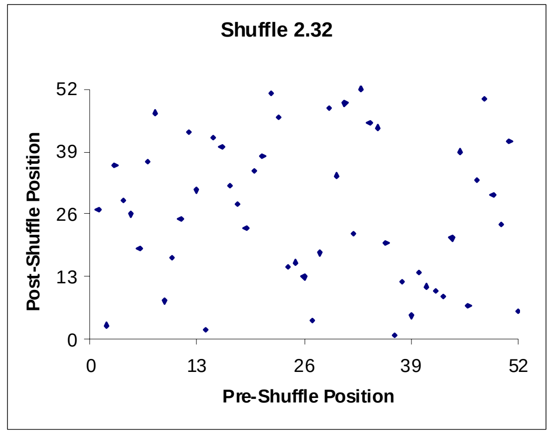
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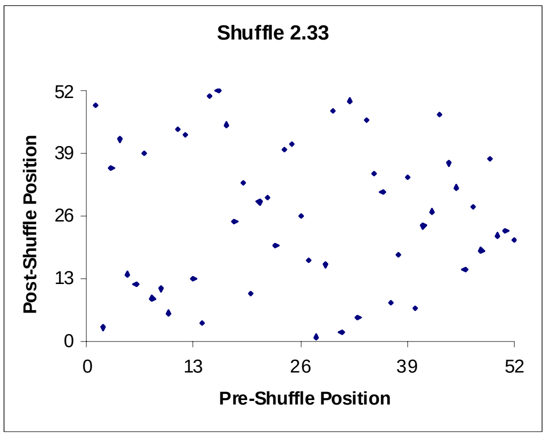
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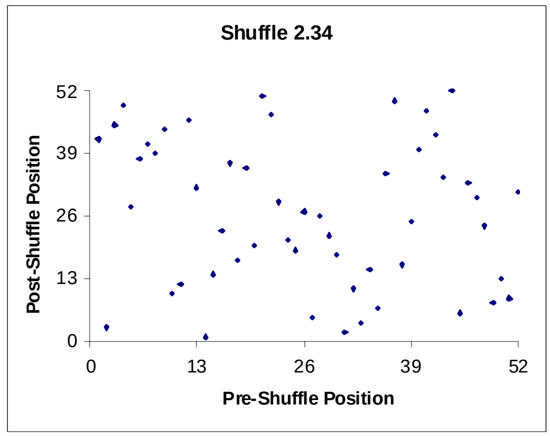
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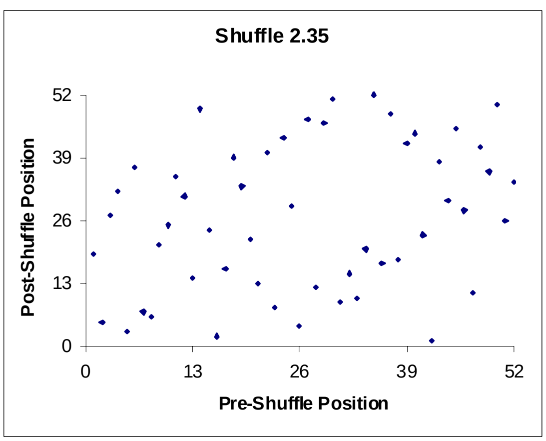
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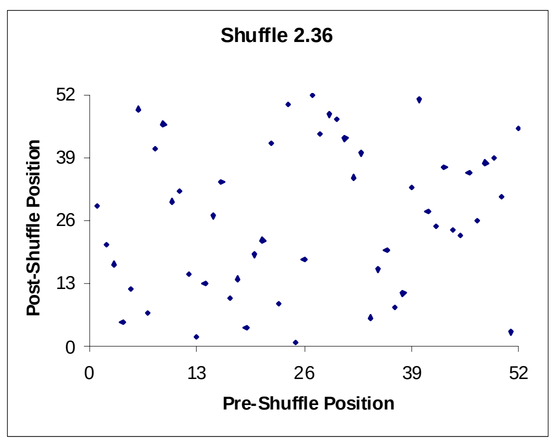
Shuffle 2.36
Post-Shuffle Position
Pre-Shuffle Position
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13
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39
52

Appendix 2 – Non-Random Plots